

$$\mathbf{h}_{v}^{(l+1)} = \sigma \left(\sum_{r \in \mathcal{R}} \sum_{u \in \mathcal{N}_{v}^{r}} \frac{1}{c_{vr}} \mathbf{W}_{r}^{(l)} \mathbf{h}_{u}^{(l)} + \mathbf{W}_{o}^{(l)} \mathbf{h}_{v}^{(l)} \right), \quad (3)$$
$$\mathbf{h}_{v} = \sum_{l=1}^{L} \alpha_{l} \mathbf{h}_{v}^{(l)}, \qquad (4)$$
where α_{l} is a trainable parameter
$$\mathbf{h}_{v} \in \mathbb{R}^{Q}. \quad \mathbf{H} \in \mathbb{R}^{|\mathcal{V}| \times Q} \quad \mathbb{R}\text{-}\mathsf{GCN}$$

where $\sigma(\cdot)$ denotes an activation function, such as ReLU, \mathcal{N}_v^r denotes the set of neighbor nodes of v under relation $r \in \mathcal{R}$, c_{vr} is a problem-specific normalization constant that can either be learned or chosen in advance (such as $c_{vr} = |\mathcal{N}_v^r|$), $\mathbf{h}_v^{(0)} = \mathbf{x}_v$ and $\mathbf{W}_r^{(l)}$ and $\mathbf{W}_o^{(l)}$ denote trainable weight matrices. 2019_EMNLP_Heterogeneous Graph Attention Networks for Semi-supervised Short Text Classification

Heterogeneous Graph Convolution

$$\textbf{GCN} \longrightarrow H^{(l+1)} = \sigma(\tilde{A} \cdot H^{(l)} \cdot W^{(l)}).$$

$$\textbf{GCN} \longrightarrow H^{(l+1)} = \sigma(\tilde{A} \cdot H^{(l)} \cdot W^{(l)}).$$

$$\textbf{M}_{ii} = \sum_{j} A'_{ij}$$

$$\tilde{A} = M^{-\frac{1}{2}}A'M^{-\frac{1}{2}}$$

HIN

$$HIN \rightarrow H^{(l+1)} = \sigma(\sum_{\tau \in \mathcal{T}} \tilde{A}_{\tau} \cdot H^{(l)}_{\tau} \cdot W^{(l)}_{\tau}),$$

Consider the fusion of multiple heterogeneous relationships
 $\mathcal{T} = \{\tau_1, \tau_2, \tau_3\}$ express different types of nodes
 $\tilde{A}_{\tau} \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}_{\tau}|}$ the submatrix of \tilde{A}
 $W^{(l)}_{\tau} \in \mathbb{R}^{q^{(l)} \times q^{(l+1)}}$

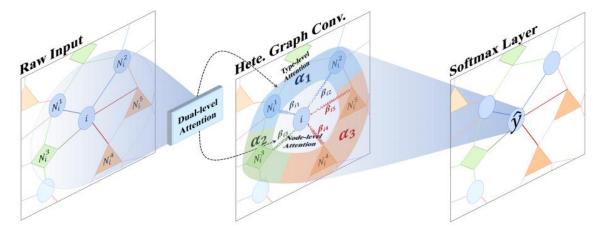
2019_EMNLP_Heterogeneous Graph Attention Networks for Semi-supervised Short Text Classification

Dual-level Attention Mechanism

Type-level Attention

$$h_{\tau} = \sum_{v'} \tilde{A}_{vv'} h_{v'}$$
$$a_{\tau} = \sigma(\mu_{\tau}^T \cdot [h_v || h_{\tau}]),$$
$$\alpha_{\tau} = \frac{\exp(a_{\tau})}{\sum_{\tau' \in \mathcal{T}} \exp(a_{\tau'})}.$$

 h_{τ} : sum of all neighbor nodes feature of node v belong to the same type τ α_{τ} : the type-level attention weights



Node-level Attention

 $b_{vv'} = \sigma(\nu^T \cdot \alpha_{\tau'}[h_v || h_{v'}]),$

$$\beta_{vv'} = \frac{\exp(b_{vv'})}{\sum_{i \in \mathcal{N}_v} \exp(b_{vi})}.$$

$$H^{(l+1)} = \sigma(\sum_{\tau \in \mathcal{T}} \mathcal{B}_{\tau} \cdot H^{(l)}_{\tau} \cdot W^{(l)}_{\tau}).$$

2021_ACL_Directed Acyclic Graph Network for Conversational Emotion Recognition

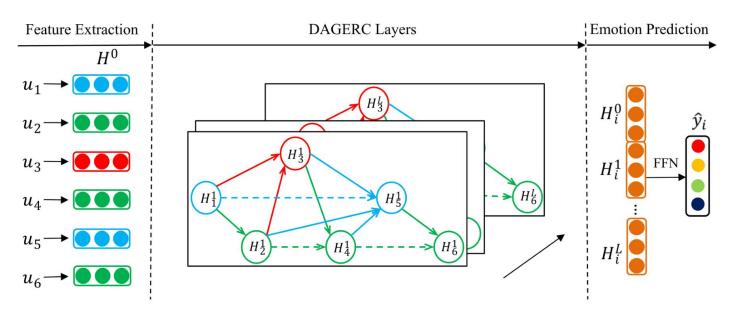


Figure 3: The framework of Directed Acyclic Graph Neural Network for ERC (DAG-ERC).

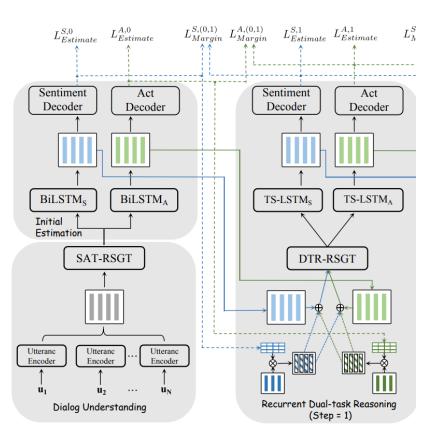
DAG-ERC Layers

$$\alpha_{ij}^{l} = \operatorname{Softmax}_{j \in \mathcal{N}_{i}}(W_{\alpha}^{l}[H_{j}^{l} \| H_{i}^{l-1}]) \quad (4)$$

$$\begin{split} M_i^l &= \sum_{j \in \mathcal{N}_i} \alpha_{ij} W_{r_{ij}}^l H_j^l, \\ \text{where } W_{r_{ij}}^l \in \{W_0^l, W_1^l\} \text{ are trainable parameters for the relation-aware transformation.} \\ \widetilde{H}_i^l &= \operatorname{GRU}_H^l(H_i^{l-1}, M_i^l), \qquad (6) \\ C_i^l &= \operatorname{GRU}_M^l(M_i^l, H_i^{l-1}). \qquad (7) \end{split}$$

$$H_i^l = \widetilde{H}_i^l + C_i^l. \tag{8}$$

2022_ACL_DARER: Dual-task Temporal Relational Recurrent Reasoning Network for Joint Dialog Sentiment Classification and Act Recognition



r_{ij}	1	2	3	4	5	6	7	8
$I_s(i) \\ I_s(j)$	1	1	-	1 2		2	_	2
$I_s(j)$ pos(i,j)		\leq	2 >	_	1	\leq	2 >	\leq

Table 2: All relation types in SATG (assume there are two speakers). $I_s(i)$ indicates the speaker node *i* is from. pos(i, j) indicates the relative position of node *i* and *j*.

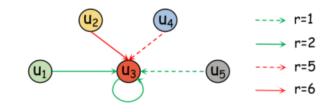


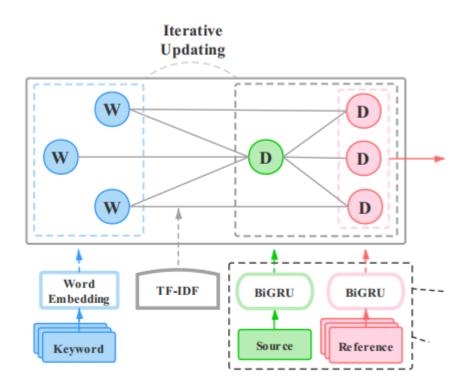
Figure 2: An example of SATG. u_1, u_3 and u_5 are from speaker 1 while u_2 and u_4 are from speaker 2. w.l.o.g, only the edges directed into u_3 node are illustrated.

Speaker-aware Temporal relation-specific graph transformations

Speaker-aware Temporal RSGT

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{R})$$
$$\hat{h}_i = W_1 h_i^0 + \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{N}_i^r} \frac{1}{|N_i^r|} W_1^r h_j^0 \qquad (1)$$
$$\hat{H} = (\hat{h}_0, \dots, \hat{h}_N)$$

AAAI_2022_Heterogeneous Graph Neural Networks for Keyphrase Generation



Heterogeneous Graph Encoder

$$z_{ij} = \text{LeakyReLU} \left(\mathbf{w}_a^T \left[\mathbf{W}_q \mathbf{h}_i; \mathbf{W}_k \mathbf{h}_j; \mathbf{e}_{ij} \right] \right)$$
$$\alpha_{ij} = \text{softmax}_j \left(z_{ij} \right) = \frac{\exp\left(z_{ij} \right)}{\sum_{k \in \mathcal{N}_i} \exp\left(z_{ik} \right)}$$
$$\mathbf{u}_i = \sigma \left(\sum_{j \in \mathcal{N}_i} \alpha_{ij} \mathbf{W}_v \mathbf{h}_j \right),$$

where \mathbf{e}_{ij} is the embedding of edge feature, α_{ij} is the attention weight between \mathbf{h}_i and \mathbf{h}_j , and \mathbf{u}_i is the aggregated feature. For simplicity, we will use GAT ($\mathbf{H}, \mathbf{H}, \mathbf{H}, \mathbf{E}$) to denote the GAT aggregating layer, where \mathbf{H} is used for query, key, and value, and \mathbf{E} is used as edge features. AAAI_2022_DDGCN: Dual Dynamic Graph Convolutional Networks for Rumor Detection on Social Media

Dual-Dynamic GCN Module

A single Dual-Static GCN unit

$$H_{ir}^{p(1)} = g\left(\hat{A}_{ir}^{p}H_{ir}^{p(0)}W_{r}^{p(0)} + b_{r}^{p(0)}\right)$$
(1)

$$H_{ir}^{p(2)} = g\left(\hat{A}_{ir}^{p}H_{ir}^{p(1)}W_{r}^{p(1)} + b_{r}^{p(1)}\right)$$
(2)

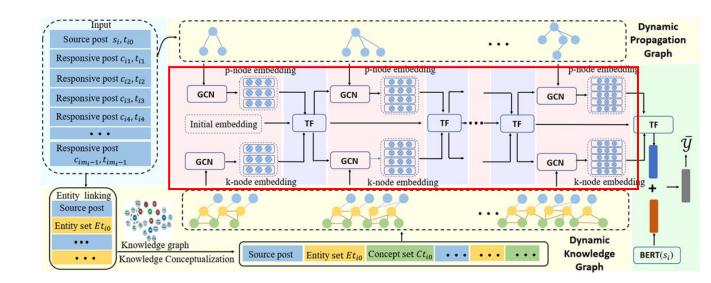
$$H_{ir}^{p^{-}} \in \mathbb{R}^{m_{i} \times F} \quad H_{ir}^{p(0)} = H_{ir}^{p}$$
$$\hat{A}_{ir}^{p} = (\tilde{D}_{ir}^{p})^{-\frac{1}{2}} \tilde{A}_{ir}^{p} (\tilde{D}_{ir}^{p})^{-\frac{1}{2}}$$
(3)

$$\tilde{H}_{ir}^{p\ (l)} = \mathbf{Concat}\left(H_{ir}^{p\ (l)}, (H_{ir}^{p\ (l-1)})^{Source}\right)$$
(4)

$$H_{ir}^{p(2)} = g\left(\hat{A}_{ir}^{p}\tilde{H}_{ir}^{p(1)}W_{r}^{p(1)} + b_{r}^{p(1)}\right)$$
(5)

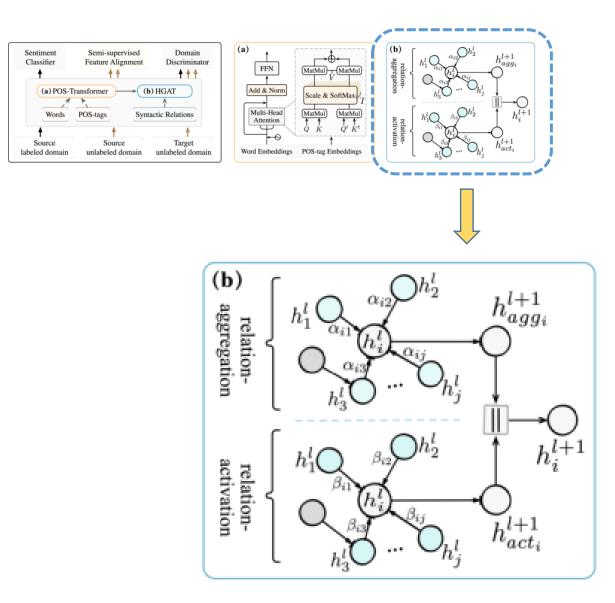
$$\tilde{H}_{ir}^{p(2)} = \mathbf{Concat}\left(H_i^{p(2)}, (H_i^{p(1)})^{Source}\right) \tag{6}$$

$$\tilde{H}_{ir}^{p\ (2)} = \mathbf{ReLU}(W_r^p \tilde{H}_{ir}^{p\ (2)} + b_r^p)$$
(7)



 $\tilde{H}_{ir}^{k\,(1)}$ and $\tilde{H}_{ir}^{k\,(2)}$ are obtained in the same manner

2022_SIGIR_Graph Adaptive Semantic Transfer for Cross-domain Sentiment Classification



$$h_{agg_i}^{l+1} = \parallel_{k=1}^{\bar{K}} \sigma(\sum_{j \in \mathcal{N}_i} \alpha_{ij}^{lk} W_{lk} h_j^l),$$
(5)

$$f_{ij}^{lk} = \sigma(a_{lk}^{T} [W_{lk} h_{i}^{l} || W_{lk} h_{j}^{l} || W_{lk} r_{ij}]),$$
(6)

$$\alpha_{ij}^{lk} = \frac{\exp\left(f_{ij}^{lk}\right)}{\sum_{j=1}^{N_i} \exp\left(f_{ij}^{lk}\right)},\tag{7}$$

$$\beta_{ij}^{lk} = \frac{\exp\left(F_{act.}(h_i^l, h_j^l)\right)}{\sum_{j=1}^{N_i} \exp\left(F_{act.}(h_i^l, h_j^l)\right)},\tag{8}$$

$$F_{act.} = \frac{\left(W_Q^{lk} h_i^l\right) \left(W_K^{lk} h_j^l + W_{Kr}^l r_{ij}\right)^T}{\sqrt{d/\bar{K}}}, \tag{9}$$

$$h_{act_i}^{l+1} = \|_{k=1}^{\bar{K}} \sigma(\sum_{j \in \mathcal{N}_i} \beta_{ij}^{lk} (W_V^{lk} h_j^l + W_{Vr}^l r_{ij})), \tag{10}$$

$$h_i^{l+1} = h_{agg_i}^{l+1} \parallel h_{act_i}^{l+1}.$$
 (11)